Immagine che contiene calligrafia, linea, Carattere, testo

Descrizione generata automaticamente*1)* Given the following graph represented by the adjacency matrix:

a) Draw the graph

b) Execute DFS from vertex and show the obtained tree

c) Same as previous point but for BFS

Immagine che contiene diagramma, linea

Descrizione generata automaticamenteSolution

*2)* Given the following weighted graph, represented by an adjacency matrix:

a) list the MST edges in the order they were determined by Kruskal’s algorithm

b) do the same using Prim’s algorithm

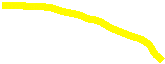
Solution

Immagine che contiene schizzo, disegno, linea

Descrizione generata automaticamenteGiven:

Immagine che contiene schizzo, disegno, linea

Descrizione generata automaticamenteWe should consider for example weights on this graph in order to make it work, for example:



Kruskal:

Prim:

Here of course in yellow, it appears the MST.

*3)* *Describe an algorithm which, given a simple graph with vertices and , determines, if it exists, a path of minimal length between and*

This comes from the theory:

* add a field .
* Modify s.t. when a is labeled
  + then ( is parent of in tree)
* Run . Check if has been visited
  + NO: then return “No path”
  + YES: starting from , follow the “parent” label, so as to build a path from to
* Complexity: where is the number of edges of connected component

*4)* *Knowing that a tree with nodes has edges, show that a connected graph with edges has edges*

Sure, here's a concise version of the explanation written in plain text without LaTeX formatting, suitable for direct copying:

A tree is a connected graph without cycles, and one of its fundamental properties is that a tree with n nodes always has n-1 edges. This property is essential for ensuring that the graph remains connected without forming any cycles.

To extend this concept to general connected graphs, consider the following:

1. Definition and Tree Property: A tree with n nodes has exactly n-1 edges. This is the minimum number of edges required to connect all the nodes without forming cycles.

2. General Connected Graphs: If a connected graph has n nodes but fewer than n-1 edges, it cannot remain connected, which contradicts the definition of a connected graph. Therefore, a connected graph with n nodes must have at least n-1 edges.

3. Adding More Edges: Adding more edges to a tree (still with n nodes) introduces cycles but maintains connectivity. Hence, any connected graph with n nodes will have at least n-1 edges. More edges than this do not disrupt connectivity but simply add cycles.

4. Visualization Through Induction (Optional):

- Base Case: A single node (n=1) has 0 edges, aligning with n-1.

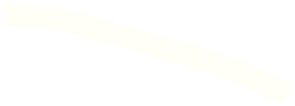
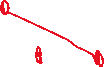
- Inductive Step: Assuming a graph with n nodes and n-1 edges is connected, adding one more node and at least one edge keeps it connected, leading to n nodes and n edges. The minimum n-1 edges ensure connectivity.

In conclusion, any connected graph with n nodes must have at least n-1 edges to ensure all nodes are connected. This minimum is achieved in tree structures, and adding more edges maintains or enhances connectivity by potentially introducing cycles.

5) *Given the following graph, show the set of edges returned by , briefly describing the algorithm*.

Immagine che contiene schizzo, disegno, linea

Descrizione generata automaticamenteAssuming once again we may be talking here about:



Remember logic and pseudocode of the approx algo:

* Choose *any* edge
* Add its endpoints to the solution
* “Remove” the covered edges
* Repeat

*Complexity*:

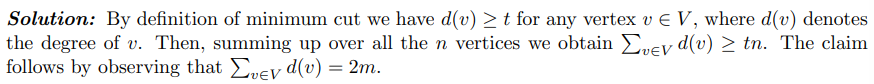
So, here we would have the part in red above.

6)

(a) *Give matching and maximal matching definition*

(b) *Show a graph in which returns a solution of cost exactly twice the optimal vertex cover*

A matching in a graph is a set of edges without common vertices, while a maximal matching is a matching which cannot be increased.

*Immagine che contiene testo, Carattere, schermata, linea

Descrizione generata automaticamenteImmagine che contiene testo, schermata, Carattere, documento

Descrizione generata automaticamente7) Show that if a graph has a minimum cut of cardinality , then it has at least edges*

